

Crustal Oscillations of Slowly Rotating Relativistic Stars

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ABSTRACT

We study low-amplitude crustal oscillations of slowly rotating relativistic stars consisting of a central fluid core and an outer thin solid crust. We estimate the effect of rotation on the torsional toroidal modes and on the interfacial and shear spheroidal modes. The results compared against the Newtonian ones for wide range of neutron star models and equations of state.

Key words: relativity – methods: numerical – stars: neutron – stars: oscillations – stars: rotation

1 INTRODUCTION

Crustal oscillations of nonrotating nonmagnetic neutron stars have been studied in Newtonian theory (Hansen & Cioffi 1980; McDermott et al. 1985; McDermott, Van Horn & Hansen 1988; Strohmayer et al. 1991; Bastrukov et al. 2007) as well as in General Relativity (Schumaker & Thorne 1983; Finn 1990; Leins 1994; Yoshida & Lee 2002; Samuelsson & Andersson 2007). These studies have been gradually extended in order to include the effects of rotation (Strohmayer 1991; Lee & Strohmayer 1996; Lee 2007a; Vavoulidis et al. 2007) or strong magnetic fields (Carroll et al. 1986; Duncan 1998; Messios, Papadopoulos & Stergioulas 2001; Piro 2005; Lee 2007a; Sotani, Kokkotas & Stergioulas 2007a; Lee 2007b; Sotani, Colaiuda & Kokkotas 2007). Most of them have been focused on torsional toroidal modes of oscillation (designated as ${}_{\ell}t_n$) and only a few studies have dealt with interfacial (${}_{\ell}i$) and shear (${}_{\ell}s_n$) spheroidal modes. There are also some recent studies of the perturbations of purely elastic stars in the general relativistic framework (Karlović, Samuelsson & Zarroug 2004; Karlović & Samuelsson 2007).

After the discovery of high-frequency quasi-periodic oscillations (QPOs) in the tails of giant flares from soft gamma-ray repeaters (SGRs) (Barat et al. 1983; Israel et al. 2005; Strohmayer & Watts 2005; Watts & Strohmayer 2006; Strohmayer & Watts 2006), special attention has been drawn to torsional toroidal oscillations of neutron star crusts. Most of the QPOs have been observed at frequencies between 18 and 155 Hz, although there have been a few at higher frequencies, for example at 625, 1840 and possibly at 718 Hz for SGR 1806-20 (Watts & Strohmayer 2007). Fundamental torsional modes could account for many of the low frequencies (Duncan 1998) while radial overtones could account for some of the higher ones (Piro 2005). However, the torsional-mode interpretation had two drawbacks: it could not explain the very low observed frequency at 18 Hz and it could not explain observed pairs of frequencies as those at 26 and 30 Hz or those at 625 and 718 Hz.

It has been soon realized that the coupling between the crust and the magnetic field would play a key-role in our attempt to explain the observed QPOs. In fact, this coupling would favour the existence of global magnetoelastic modes of oscillation rather than pure elastic modes confined in the crust (Levin 2006). Moreover, it has been argued that these modes should decay on a short timescale because of the presence of a magnetohydrodynamical continuum in the core and only specific QPOs could be long-lived (Glampedakis, Samuelsson & Andersson 2006; Sotani et al. 2006; Lee 2007a; Levin 2007; Sotani, Kokkotas & Stergioulas 2007b).

After a catastrophic reconfiguration of the stellar magnetic field, axial-type torsional and Alfvén oscillations should be the most easily excited as polar-type oscillations would have to overcome strong restoring forces. However, the presence of a magnetic field would inevitably couple these oscillations with polar-type ones characterized by interfacial (i-), shear (s-), pressure-restored (f- and p-), gravity (g-) and polar-type magnetoelastic modes; stellar rotation should do more or less the same. These polar-type modes would involve density variations and could be relevant for gravitational-wave emission

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(Abbott et al. 2007, LIGO scientific collaboration). The possible association of polar-type modes with the observed QPOs has been hinted by Piro (2005); Watts & Strohmayer (2006) but further investigation is definitely needed.

In Section 2 we derive the equations that describe linear spheroidal and toroidal oscillations of slowly rotating relativistic stars possessing a crust. In most of them, a parameter ε indicates the presence of a relativistic term. Therefore $\varepsilon \rightarrow 0$ becomes our standard tool to recover the already known Newtonian equations for these oscillations (Strohmayer 1991; Lee & Strohmayer 1996). Furthermore, the nonrotating parts of these relativistic equations are easily comparable with the equations given in Yoshida & Lee (2002). In Section 2 we derive the perturbation equations for the solid-crust region (2.1), we review those for the fluid-core region (2.2) and we present the necessary physical boundary and jump conditions (2.3). In Section 3 we present our numerical results for a set of neutron star models with different equations of state (EoS) and different bulk properties (3.1), first revisiting torsional toroidal modes (3.2) and then focusing on interfacial and shear spheroidal modes (3.3). In Section 4 we summarize and discuss our results.

2 FORMULATION

We consider a slowly rotating relativistic, strain free, star described by the metric:

$$ds^2 = -e^{2\nu} dt^2 + e^{2\lambda} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 - 2\omega r^2 \sin^2 \theta dt d\phi, \quad (1)$$

where ν , λ and ω are functions of the radial coordinate r . These functions are solutions of the Tolman-Oppenheimer-Volkoff (TOV) equations:

$$e^{2\lambda} = \left(1 - \varepsilon \frac{2M(r)}{r}\right)^{-1}, \quad \frac{dM(r)}{dr} = 4\pi r^2 \rho, \quad (2)$$

$$\frac{d\nu}{dr} = e^{2\lambda} \left(4\pi r \varepsilon p + \frac{M(r)}{r^2}\right), \quad \frac{dp}{dr} = -(\rho + \varepsilon p) \frac{d\nu}{dr}, \quad (3)$$

$$\frac{d\lambda}{dr} = \varepsilon e^{2\lambda} \left(4\pi r \rho - \frac{M(r)}{r^2}\right), \quad (4)$$

and of one more equation describing the dragging of the inertial frames of reference (Hartle 1967):

$$\frac{d^2 \varpi}{dr^2} - \left(\frac{d\nu}{dr} + \frac{d\lambda}{dr} - \frac{4}{r}\right) \frac{d\varpi}{dr} - 16\pi e^{2\lambda} (\rho + p) \varpi = 0, \quad (5)$$

where $\varpi := \Omega - \omega$, ρ and p are the energy density and the pressure, respectively, $M(r)$ is the mass inside radius r and Ω is the stellar rotational frequency. In our general-relativistic approach, the parameter ε equals to 1. Its presence aims to point out the Newtonian limit where $\varepsilon \rightarrow 0$. Then, according to equations (2)-(4), $\lambda \rightarrow 0$, $d\nu/dr \rightarrow M(r)/r^2$, $dp/dr \rightarrow -\rho d\nu/dr$ and $d\lambda/dr \rightarrow 0$. Furthermore, in the Newtonian limit, $\omega \rightarrow 0$ or, equivalently, $\varpi \rightarrow \Omega$ and $d\varpi/dr \rightarrow 0$.

The pulsation equations come from the linear perturbation of the energy-momentum conservation law $\delta(\nabla^\beta T_{\alpha\beta}) = 0$ where:

$$T_{\alpha\beta} = (\rho + p) u_\alpha u_\beta + p g_{\alpha\beta} - 2\mu S_{\alpha\beta}. \quad (6)$$

The components of the perturbed four-velocity are given by the relation $\delta u^\alpha = \mathcal{L}_u \xi^\alpha$ where ξ^α is the displacement vector while the shear tensor $S_{\alpha\beta}$ is given by the relation $\sigma_{\alpha\beta} = \mathcal{L}_u S_{\alpha\beta}$ where $\sigma_{\alpha\beta}$ is the rate of shear tensor. Namely $\sigma_{\alpha\beta}$ is the Lie derivative of the shear tensor along the world lines (Carter & Quintana 1972) and is calculated by the equation:

$$\sigma_{\alpha\beta} = \frac{1}{2} (P_\beta^\gamma \nabla_\gamma u_\alpha + P_\alpha^\gamma \nabla_\gamma u_\beta) - \frac{1}{3} P_{\alpha\beta} \nabla_\gamma u^\gamma, \quad (7)$$

where $P_{\alpha\beta}$ is the projection tensor:

$$P_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta. \quad (8)$$

It is implied that we work in the Cowling approximation as we neglect the perturbed Einstein equations $\delta G_{\alpha\beta} = 8\pi \delta T_{\alpha\beta}$ and, additionally, set all metric perturbations equal to zero in our equations. The Cowling approximation is typically very good for toroidal type of oscillations i.e. t -modes, r -modes etc, while typically for polar type of perturbations the error can be of the order of 10-20% especially for the fundamental pressure mode the f -mode. For, the other type of modes, like g -modes, higher p -modes and s -modes one does not expect deviations larger than 3-5%.

We choose to work in a corotating reference frame where $\delta u^\alpha = e^{-\nu} \partial \xi^\alpha / \partial t$, $\sigma_{\alpha\beta} = e^{-\nu} \partial S_{\alpha\beta} / \partial t$. Using equations (6)-(8), the energy-momentum conservation law yields three second-order partial differential equations for the three components of the displacement vector $\xi^r = \xi^r(t, r, \theta, \phi)$, $\xi^\theta = \xi^\theta(t, r, \theta, \phi)$ and $\xi^\phi = \xi^\phi(t, r, \theta, \phi)$. These equations are the relativistic version of the Newtonian equations (35)-(39) of Strohmayer (1991) and their explicit form is given in the Appendix A.

When we focus on spheroidal modes, we can write for the displacement vector:

$$\xi^i = \left[rS, H \frac{\partial}{\partial \theta}, H \frac{1}{\sin^2 \theta} \frac{\partial}{\partial \phi} \right] Y_{\ell m} e^{i\sigma t}, \quad (9)$$

while when we focus on toroidal modes, we can write:

$$\xi^i = \left[0, T \frac{1}{\sin \theta} \frac{\partial}{\partial \phi}, -T \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right] Y_{\ell m} e^{i\sigma t}. \quad (10)$$

Working in the slow-rotation approximation (Kojima 1992; Stavridis & Kokkotas 2005), we eventually obtain three second-order ordinary differential equations for the functions $S = S(r)$, $H = H(r)$ and $T = T(r)$. The equations for S and H are quite lengthy and we do not show them here. However, we mention that they are the relativistic version of the equations used in Lee & Strohmayer (1996). On the other hand, the equation for the toroidal radial function T , being shorter, reads (Vavoulidis et al. 2007):

$$-\sigma^2 T = v_s^2 e^{2\varepsilon(\nu-\lambda)} \left[\frac{d^2 T}{dr^2} + \left(\frac{4}{r} + \varepsilon \frac{d\nu}{dr} - \varepsilon \frac{d\lambda}{dr} + \frac{1}{\mu} \frac{d\mu}{dr} \right) \frac{dT}{dr} - e^{2\varepsilon\lambda} \frac{\Lambda - 2}{r^2} T \right] - 2m\sigma\varpi \left[\frac{1}{\Lambda} + \varepsilon v_s^2 \left(1 - \frac{2}{\Lambda} \right) \right] T, \quad (11)$$

where $v_s^2 := \mu/(\rho + \varepsilon p)$ is the speed of shear waves and $\Lambda := \ell(\ell + 1)$.

We then expand the frequency and the displacement functions in power series:

$$\sigma = \sum_{j=0}^{\infty} \eta^j \sigma_j = \sigma_0 + \eta \sigma_1 + \mathcal{O}(\eta^2), \quad \xi^i = \sum_{j=0}^{\infty} \eta^j \xi^{i,j} = \xi^{i,0} + \eta \xi^{i,1} + \mathcal{O}(\eta^2), \quad (12)$$

where η is an auxiliary expansion parameter and we insert these expansions into the main perturbation equations, e.g. equation (11) above. In this way we split each equation in two, one zeroth-order in Ω which is taken collecting all η^0 terms and one first-order in Ω which is taken collecting all η^1 terms. For example, equation (11), to zeroth order in rotation, gives the equation:

$$-\sigma_0^2 T^0 = v_s^2 e^{2\varepsilon(\nu-\lambda)} \left[\frac{d^2 T^0}{dr^2} + \left(\frac{4}{r} + \varepsilon \frac{d\nu}{dr} - \varepsilon \frac{d\lambda}{dr} + \frac{1}{\mu} \frac{d\mu}{dr} \right) \frac{dT^0}{dr} - e^{2\varepsilon\lambda} \frac{\Lambda - 2}{r^2} T^0 \right], \quad (13)$$

which, when supplied with the appropriate boundary conditions, determines the zeroth-order eigenfrequency, σ_0 , and the zeroth-order eigenfunction, T^0 . Then, to first order in rotation, equation (11) gives another equation:

$$\begin{aligned} -\sigma_0^2 T^1 - 2\sigma_0 \sigma_1 T^0 &= v_s^2 e^{2\varepsilon(\nu-\lambda)} \left[\frac{d^2 T^1}{dr^2} + \left(\frac{4}{r} + \varepsilon \frac{d\nu}{dr} - \varepsilon \frac{d\lambda}{dr} + \frac{1}{\mu} \frac{d\mu}{dr} \right) \frac{dT^1}{dr} - e^{2\varepsilon\lambda} \frac{\Lambda - 2}{r^2} T^1 \right] \\ &- 2m\sigma_0 \varpi \left[\frac{1}{\Lambda} + \varepsilon v_s^2 \left(1 - \frac{2}{\Lambda} \right) \right] T^0, \end{aligned} \quad (14)$$

which determines the first-order rotational corrections of the eigenfrequency, σ_1 , and of the eigenfunction, T^1 , using the known zeroth-order quantities σ_0 and T^0 , already calculated by equation (13).

2.1 Solid crust

To describe the oscillations in the solid-crust region, we use the following functions:

$$z_1^j = S^j, \quad (15)$$

$$z_2^j = 2\alpha_1 e^{-\varepsilon\lambda} \frac{d}{dr} \left(r e^{\varepsilon\lambda} S^j \right) + \left(\Gamma - \frac{2}{3} \alpha_1 \right) \left\{ \frac{e^{-\varepsilon\lambda}}{r^2} \frac{d}{dr} \left(r^3 e^{\varepsilon\lambda} S^j \right) - \ell(\ell + 1) H^j \right\}, \quad (16)$$

$$z_3^j = H^j, \quad (17)$$

$$z_4^j = \alpha_1 \left(e^{-2\varepsilon\lambda} r \frac{dH^j}{dr} + S^j \right), \quad (18)$$

$$z_5^j = T^j, \quad (19)$$

$$z_6^j = \alpha_1 e^{-2\varepsilon\lambda} r \frac{dT^j}{dr}. \quad (20)$$

This set of functions is the set (31)-(36) of Yoshida & Lee (2002) and it reduces to the set of functions (53),(16),(15) of Lee & Strohmayer (1996) in the Newtonian limit, $\varepsilon \rightarrow 0$.

Using these functions, we can recast our three main perturbation equations into a system of six first-order ordinary differential equations where four of them are describing the spheroidal perturbations while the other two are describing the toroidal perturbations. To zeroth order in Ω ($j = 0$), this system has the form:

$$r \frac{dz_1^0}{dr} = - \left(1 + 2 \frac{\alpha_2}{\alpha_3} + \varepsilon U_2 \right) z_1^0 + \frac{1}{\alpha_3} z_2^0 + \frac{\alpha_2}{\alpha_3} \ell(\ell + 1) z_3^0, \quad (21)$$

$$r \frac{dz_2^0}{dr} = \left\{ \left(-3 - \varepsilon U_2 + U_1 - e^{2\varepsilon\lambda} c_1 \bar{\sigma}_0^2 \right) V_1 + 4 \frac{\alpha_1}{\alpha_3} (3\alpha_2 + 2\alpha_1) \right\} z_1^0 + \left(V_2 - 4 \frac{\alpha_1}{\alpha_3} \right) z_2^0 \\ + \left\{ V_1 - 2\alpha_1 \left(1 + 2 \frac{\alpha_2}{\alpha_3} \right) \right\} \ell(\ell+1) z_3^0 + e^{2\varepsilon\lambda} \ell(\ell+1) z_4^0, \quad (22)$$

$$r \frac{dz_3^0}{dr} = -e^{2\varepsilon\lambda} z_1^0 + \frac{e^{2\varepsilon\lambda}}{\alpha_1} z_4^0, \quad (23)$$

$$r \frac{dz_4^0}{dr} = - \left(-V_1 + 6\Gamma \frac{\alpha_1}{\alpha_3} \right) z_1^0 - \frac{\alpha_2}{\alpha_3} z_2^0 - \left\{ c_1 \bar{\sigma}_0^2 V_1 + 2\alpha_1 - 2 \frac{\alpha_1}{\alpha_3} (\alpha_2 + \alpha_3) \ell(\ell+1) \right\} z_3^0 - (3 + \varepsilon U_2 - V_2) z_4^0, \quad (24)$$

$$r \frac{dz_5^0}{dr} = \frac{e^{2\varepsilon\lambda}}{\alpha_1} z_6^0, \quad (25)$$

$$r \frac{dz_6^0}{dr} = - \left\{ c_1 \bar{\sigma}_0^2 V_1 - \alpha_1 (\ell-1)(\ell+2) \right\} z_5^0 - (3 + \varepsilon U_2 - V_2) z_6^0. \quad (26)$$

This set of equations is the set (25)-(30) of Yoshida & Lee (2002) and, in the Newtonian limit, reduces to the sets of equations (54)-(57) and (68)-(69) of Lee & Strohmayer (1996). The first four equations give the zeroth-order eigenfrequency σ_0 and eigenfunctions $z_1^0, z_2^0, z_3^0, z_4^0$ for the interfacial and shear spheroidal oscillations while the last two equations give σ_0, z_5^0 and z_6^0 for the torsional toroidal oscillations.

As in Yoshida & Lee (2002), the functions that appear in equations (21)-(26) are:

$$\alpha_1 = \frac{\mu}{p}, \quad \alpha_2 = \Gamma - \frac{2}{3}\alpha_1, \quad \alpha_3 = \Gamma + \frac{4}{3}\alpha_1, \quad (27)$$

$$V_1 = \frac{\rho + \varepsilon p}{p} r \frac{d\nu}{dr}, \quad V_2 = \frac{\rho}{p} r \frac{d\nu}{dr}, \quad (28)$$

$$U_1 = \left(\frac{d\nu}{dr} \right)^{-1} \frac{d}{dr} \left(r \frac{d\nu}{dr} \right), \quad U_2 = \varepsilon r \frac{d\lambda}{dr}, \quad (29)$$

$$c_1 = \frac{M}{R^3} r e^{-2\varepsilon\nu} \left(\frac{d\nu}{dr} \right)^{-1}. \quad (30)$$

In the Newtonian limit, $\varepsilon \rightarrow 0$ and $d\nu/dr \rightarrow g$ where $g = M(r)/r^2$ is the gravitational acceleration with $dp/dr \rightarrow -\rho g$. Therefore, the functions (28)-(30) reduce to:

$$V_1, V_2 \rightarrow \frac{\rho}{p} r g = - \frac{d \ln p}{d \ln r} = V, \quad (31)$$

$$U_1 \rightarrow g^{-1} \frac{d}{dr} (r g) = \frac{r^2}{M(r)} \frac{d}{dr} \left(\frac{M(r)}{r} \right) = \frac{d \ln M(r)}{d \ln r} - 1 = U - 1, \quad (32)$$

$$c_1 \rightarrow \frac{M}{R^3} r g^{-1} = \left(\frac{r}{R} \right)^3 \frac{M}{M(r)}. \quad (33)$$

The functions V, U and c_1 are well-known from the Newtonian theory, see e.g. the set of relations (35) of Lee & Strohmayer (1996) or the set of relations (15) of McDermott, Van Horn & Hansen (1988).

To first order in Ω ($j=1$), our system of equations has the form:

$$r \frac{dz_1^1}{dr} = - \left(1 + 2 \frac{\alpha_2}{\alpha_3} + \varepsilon U_2 \right) z_1^1 + \frac{1}{\alpha_3} z_2^1 + \frac{\alpha_2}{\alpha_3} \ell(\ell+1) z_3^1, \quad (34)$$

$$r \frac{dz_2^1}{dr} = \left\{ \left(-3 - \varepsilon U_2 + U_1 - e^{2\varepsilon\lambda} c_1 \bar{\sigma}_0^2 \right) V_1 + 4 \frac{\alpha_1}{\alpha_3} (3\alpha_2 + 2\alpha_1) \right\} z_1^1 + \left(V_2 - 4 \frac{\alpha_1}{\alpha_3} \right) z_2^1 \\ + \left\{ V_1 - 2\alpha_1 \left(1 + 2 \frac{\alpha_2}{\alpha_3} \right) \right\} \ell(\ell+1) z_3^1 + e^{2\varepsilon\lambda} \ell(\ell+1) z_4^1 \\ + \left\{ -2e^{2\varepsilon\lambda} c_1 \bar{\sigma}_0 \bar{\sigma}_1 V_1 + \varepsilon \mathcal{A} \right\} z_1^0 + \{ 2m c_1 \bar{\sigma}_0 \bar{\omega} V_1 + \varepsilon \mathcal{B} \} z_3^0 + \varepsilon \mathcal{C} z_4^0, \quad (35)$$

$$r \frac{dz_3^1}{dr} = -e^{2\varepsilon\lambda} z_1^1 + \frac{e^{2\varepsilon\lambda}}{\alpha_1} z_4^1, \quad (36)$$

$$r \frac{dz_4^1}{dr} = - \left(-V_1 + 6\Gamma \frac{\alpha_1}{\alpha_3} \right) z_1^1 - \frac{\alpha_2}{\alpha_3} z_2^1 - \left\{ c_1 \bar{\sigma}_0^2 V_1 + 2\alpha_1 - 2 \frac{\alpha_1}{\alpha_3} (\alpha_2 + \alpha_3) \ell(\ell+1) \right\} z_3^1 - (3 + \varepsilon U_2 - V_2) z_4^1 \\ + \left\{ \frac{2m c_1 \bar{\sigma}_0 \bar{\omega} V_1}{\ell(\ell+1)} + \varepsilon \mathcal{D} \right\} z_1^0 + \varepsilon \mathcal{E} z_2^0 + \left\{ -2c_1 \bar{\sigma}_0 \bar{\sigma}_1 V_1 + \frac{2m c_1 \bar{\sigma}_0 \bar{\omega} V_1}{\ell(\ell+1)} + \varepsilon \mathcal{F} \right\} z_3^0, \quad (37)$$

$$r \frac{dz_5^1}{dr} = \frac{e^{2\varepsilon\lambda}}{\alpha_1} z_6^1, \quad (38)$$

$$r \frac{dz_6^1}{dr} = - \left\{ c_1 \bar{\sigma}_0^2 V_1 - \alpha_1 (\ell-1)(\ell+2) \right\} z_5^1 - (3 + \varepsilon U_2 - V_2) z_6^1$$

$$+ \left\{ 2c_1 \bar{\sigma}_0 V_1 \left[-\bar{\sigma}_1 + \frac{m\bar{\omega}}{\ell(\ell+1)} \right] + \varepsilon \mathcal{G} \right\} z_5^0, \quad (39)$$

where:

$$\mathcal{A} = e^{-2\nu+2\lambda} (\alpha_1 - \alpha_2) r^2 m \sigma_0 \varpi, \quad (40)$$

$$\mathcal{B} = e^{-2\nu} \left[\left(\frac{\rho}{p} + 1 + \alpha_2 \right) r^3 m \sigma_0 \frac{d\varpi}{dr} - \left[\left(\left(\frac{\rho}{p} + 1 \right) (\Gamma + 1) + \alpha_2 \right) r \frac{d\nu}{dr} + \frac{2}{3} \frac{r}{p} \frac{d\mu}{dr} + 2(\alpha_1 - \alpha_2) \right] r^2 m \sigma_0 \varpi \right], \quad (41)$$

$$\mathcal{C} = e^{-2\nu+2\lambda} \left(\frac{\alpha_3}{\alpha_1} - 1 \right) r^2 m \sigma_0 \varpi, \quad (42)$$

$$\mathcal{D} = e^{-2\nu} \left[\left(\frac{\rho}{p} + 1 - \alpha_1 \right) \frac{r^3 m \sigma_0}{\ell(\ell+1)} \frac{d\varpi}{dr} - \left[\left(\frac{\rho}{p} + 1 - \alpha_1 \right) r \frac{d\nu}{dr} + \frac{r}{p} \frac{d\mu}{dr} + 2 \left(\alpha_1 - \alpha_2 + \alpha_3 + \frac{\alpha_1 \alpha_2}{\alpha_3} \right) \right] \frac{r^2 m \sigma_0 \varpi}{\ell(\ell+1)} \right], \quad (43)$$

$$\mathcal{E} = e^{-2\nu} \left(\frac{\alpha_1}{\alpha_3} - 1 \right) \frac{r^2 m \sigma_0 \varpi}{\ell(\ell+1)}, \quad (44)$$

$$\mathcal{F} = e^{-2\nu} \left(-4\alpha_1 + \left(2\alpha_3 - \alpha_2 + \frac{\alpha_1 \alpha_2}{\alpha_3} \right) \ell(\ell+1) \right) \frac{r^2 m \sigma_0 \varpi}{\ell(\ell+1)}, \quad (45)$$

$$\mathcal{G} = e^{-2\nu} 2\alpha_1 (\ell-1)(\ell+2) \frac{r^2 m \sigma_0 \varpi}{\ell(\ell+1)}. \quad (46)$$

From equations (34)-(39) we get the first-order rotational correction of the eigenfrequency, σ_1 , and the first-order rotational corrections of the eigenfunctions, z_i^1 ($i = 1 \dots 6$). Again, the first four of them refer to the interfacial and shear spheroidal modes while the last two refer to the torsional toroidal modes. Bars over σ_0, σ_1 and ϖ indicate dimensionless quantities, scaled by $\sqrt{M/R^3}$; for example $\bar{\sigma}_0 := \sigma_0/\sqrt{M/R^3}$. This set of equations reduces to the sets of equations (58)-(61) and (70)-(71) of Lee & Strohmayer (1996) in the Newtonian limit.

2.2 Fluid core

To describe the oscillations in the fluid-core region, we use the following functions:

$$y_1^j = S^j, \quad (47)$$

$$y_2^j = \left(r \frac{d\nu}{dr} \right)^{-1} \frac{\delta p^j}{\rho + \varepsilon p} \stackrel{\varepsilon \rightarrow 0}{=} \frac{\delta p^j}{g r \rho}. \quad (48)$$

The zeroth and first-order spheroidal radial functions H^0 and H^1 are given by the relations:

$$H^0 = \frac{y_2^0}{c_1 \bar{\sigma}_0^2}, \quad (49)$$

$$H^1 = \frac{y_2^1}{c_1 \bar{\sigma}_0^2} + \left\{ \frac{2m\bar{\omega}}{\bar{\sigma}_0} + \varepsilon \mathcal{H} \right\} \frac{y_1^0}{\ell(\ell+1)} + \left\{ \frac{2m\bar{\omega}}{\bar{\sigma}_0} \left(\frac{1}{\ell(\ell+1)} - \frac{\bar{\sigma}_1}{m\bar{\omega}} \right) + \varepsilon \frac{1}{\ell(\ell+1)} \mathcal{I} \right\} \frac{y_2^0}{c_1 \bar{\sigma}_0^2}, \quad (50)$$

cf. relations (44) of Yoshida & Lee (2002) for nonrotating relativistic stars and relations (81),(86)-(87) of Lee & Strohmayer (1996) for rotating Newtonian stars.

To zeroth order in Ω ($j = 0$), the system of equations (A1)-(A3) with $\mu = 0$ can be recasted into a system of two first-order ordinary differential equations for the functions y_1^0 and y_2^0 :

$$r \frac{dy_1^0}{dr} = - \left(3 - \frac{V_1}{\Gamma} + \varepsilon U_2 \right) y_1^0 - \left(\frac{V_1}{\Gamma} - \frac{\ell(\ell+1)}{c_1 \bar{\sigma}_0^2} \right) y_2^0, \quad (51)$$

$$r \frac{dy_2^0}{dr} = \left(e^{2\varepsilon\lambda} c_1 \bar{\sigma}_0^2 + r A_r \right) y_1^0 - (U_1 + r A_r) y_2^0, \quad (52)$$

where A_r is the relativistic Schwarzschild discriminant:

$$A_r = \frac{1}{\rho + \varepsilon p} \frac{d\rho}{dr} - \frac{1}{\Gamma p} \frac{dp}{dr}. \quad (53)$$

Similarly, to first order in Ω ($j = 1$), our system for the functions y_1^1 and y_2^1 is the following:

$$\begin{aligned} r \frac{dy_1^1}{dr} &= - \left(3 - \frac{V_1}{\Gamma} + \varepsilon U_2 \right) y_1^1 - \left(\frac{V_1}{\Gamma} - \frac{\ell(\ell+1)}{c_1 \bar{\sigma}_0^2} \right) y_2^1 \\ &+ \left\{ \frac{2m\bar{\omega}}{\bar{\sigma}_0} + \varepsilon \mathcal{H} \right\} y_1^0 + \left\{ \left[\frac{2m\bar{\omega}}{\bar{\sigma}_0} - \ell(\ell+1) \frac{2\bar{\sigma}_1}{\bar{\sigma}_0} \right] + 2\varepsilon \mathcal{I} \right\} \frac{y_2^0}{c_1 \bar{\sigma}_0^2}, \\ r \frac{dy_2^1}{dr} &= \left(e^{2\varepsilon\lambda} c_1 \bar{\sigma}_0^2 + r A_r \right) y_1^1 - (U_1 + r A_r) y_2^1 \end{aligned} \quad (54)$$

$$+ 2e^{2\varepsilon\lambda} c_1 \bar{\sigma}_0 \bar{\sigma}_1 y_1^0 - \left\{ \frac{2m\bar{\omega}}{\bar{\sigma}_0} + \varepsilon\mathcal{H} \right\} y_2^0, \quad (55)$$

where:

$$\mathcal{H} = \left(\frac{d\varpi}{dr} - 2 \frac{d\nu}{dr} \varpi \right) \frac{rm}{\sigma_0}, \quad (56)$$

$$\mathcal{I} = e^{-2\nu} r^2 m \sigma_0 \varpi. \quad (57)$$

In the Newtonian limit $\varepsilon \rightarrow 0$, $V_1 \rightarrow V$, $\varpi \rightarrow \Omega$ and equations (51)-(52) and (54)-(55) reduce to equations (82)-(83) and (84)-(85) of Lee & Strohmayer (1996), respectively. In the relativistic nonrotating limit, $\Omega \rightarrow 0$, we are reduced to equations (42)-(43) of Yoshida & Lee (2002).

2.3 Boundary and normalization conditions

At the stellar center ($r = 0$), the eigenfunctions y_1^j and y_2^j must be regular. By expanding them in appropriate series, $y_i = \sum_0^n \alpha_{i,n} r^n$ and carrying out some algebraic manipulations, we find:

$$c_1 \bar{\sigma}_0^2 y_1^j - \ell y_2^j + F^j = 0, \quad (58)$$

where:

$$F^0 = 0, \quad (59)$$

$$F^1 = \frac{2m\varpi}{\sigma_0} \left(\frac{\sigma_1}{m\varpi} - \frac{1}{\ell} \right) c_1 \bar{\sigma}_0^2 y_1^0. \quad (60)$$

At the stellar surface ($r = R$), the Lagrangian perturbation of the pressure must vanish ($\Delta p = 0$). This eventually leads to a simple relation between y_1^j and y_2^j , i.e.:

$$y_1^j - y_2^j = 0. \quad (61)$$

At the fluid-solid interfaces, we require continuity of the tractions. This means the jump conditions:

$$z_1^j = y_1^j, \quad (62)$$

$$z_2^j = V_1 (y_1^j - y_2^j), \quad (63)$$

$$z_4^j = 0, \quad (64)$$

for the spheroidal modes, and:

$$z_6^j = 0, \quad (65)$$

for the toroidal modes.

Finally, we normalize our zeroth and first-order eigenfunctions by imposing the conditions:

$$y_1^0 = 1, \quad (66)$$

$$y_1^1 = 0, \quad (67)$$

respectively, at the stellar surface.

3 RESULTS

3.1 Neutron star models

We work with a set of 34 realistic neutron star models: we combine EoS A (Pandharipande 1971), WFF3 (Wiringa, Fiks & Fabrocini 1988), APR (Akmal, Pandharipande & Ravenhall 1998) or L (Pandharipande & Smith 1975) for the fluid core with EoS DH (Douchin & Haensel 2001) or NV (Negele & Vautherin 1973) for the solid crust and, for each combination, we construct a sequence of models, beginning from a low-mass model of $1.4M_\odot$ and reaching, in increments of $0.2M_\odot$, the maximum-mass model allowed by that EoS. That maximum-mass model is $\gtrsim 1.6, 1.8, 2.2$ and $2.6M_\odot$ for EoS A, WFF3, APR and L, respectively. Further details about this set of neutron star models can be found in Sotani, Kokkotas & Stergioulas (2007a) and in references therein. Here, we add the following pair of fitting formulas:

$$\ln(\Delta r/R) \simeq -7.95(\pm 0.04) M/R - 1.28(\pm 0.01), \quad \text{for the DH crustal EoS,} \quad (68)$$

$$\ln(\Delta r/R) \simeq -7.84(\pm 0.03) M/R - 1.04(\pm 0.01), \quad \text{for the NV crustal EoS,} \quad (69)$$

which relate stellar compactness M/R with relative crust thickness $\Delta r/R$. It is obvious that this formulae are valid for typical neutron stars with a fluid core and a crust. In the limit where $M/R \rightarrow 0$ the star “solidifies” or else the crust equation of state is valid for the whole star and the above formulae do not reach the correct limit which is just 1. We should point out that a semi-analytic formula has already been derived by Samuelsson & Andersson (2007), this formula is valid for a wide range of stellar compactness and has the correct $M/R \rightarrow 0$ limit. In this formula [eq (B6)] there is a free parameter α which should be fixed by the equation of state. Samuelsson & Andersson (2007), for the crust EoS DH, find $\alpha \approx 0.019$ for $\Gamma = 4/3$ polytropes and $\alpha \approx 0.023$ for the set of realistic equations of state for the fluid core that they have used. For our stellar models we find $\alpha \approx 0.020$ for the crust EoS DH and $\alpha = 0.026$ for the crust EoS NV.

3.2 Torsional modes

The Newtonian first-order rotational corrections of the eigenfrequencies of the torsional modes are shown in the formula of Strohmayer (1991):

$$\sigma = \sigma_0 + \sigma_1 = \sigma_0 + m\Omega C_{1,\text{New}} = \sigma_0 + \frac{m\Omega}{\ell(\ell+1)}, \quad (70)$$

which indicates that $C_{1,\text{New}} := \sigma_1/m\Omega$ takes the value $1/[\ell(\ell+1)]$ independently of the structure of the star. For example, $C_{1,\text{New}} = 0.167$ for $\ell = 2$, $C_{1,\text{New}} = 0.083$ for $\ell = 3$, $C_{1,\text{New}} = 0.05$ for $\ell = 4$ and so on. Moreover, the Newtonian first-order rotational corrections of the eigenfunctions of the torsional modes can be set equal to zero, $T^1 = 0$, cf. Strohmayer (1991) and Lee & Strohmayer (1996). The relativistic results can be drawn from Tables 1 and 2 and from Figure 1. The relativistic first-order rotational corrections of the eigenfrequencies of the torsional modes, as measured from a rotating observer, are shown in the formula:

$$\sigma = \sigma_0 + \sigma_1 = \sigma_0 + m\Omega C_{1,\text{rel}}. \quad (71)$$

Then, the rotating observer compares the Newtonian and the relativistic first-order rotational corrections defining $rc_r := 1 - C_{1,\text{rel}}/C_{1,\text{New}}$. On the other hand, an inertial observer measures:

$$\sigma = \sigma_0 - m\Omega + m\Omega C_{1,\text{New}} = \sigma_0 + m\Omega C'_{1,\text{New}}, \quad (72)$$

$$\sigma = \sigma_0 - m\Omega + m\Omega C_{1,\text{rel}} = \sigma_0 + m\Omega C'_{1,\text{rel}}, \quad (73)$$

in the Newtonian and in the relativistic case, respectively, and he defines $rc_i := 1 - C'_{1,\text{rel}}/C'_{1,\text{New}}$. It is easy to check that rc_r and rc_i obey the simple relation:

$$rc_i = -\frac{rc_r}{1/C_{1,\text{New}} - 1}. \quad (74)$$

In Table 1 we list the eigenfrequencies σ_0 and the relativistic first-order rotational corrections $C_{1,\text{rel}}$ for the fundamental $\ell = 2, 3$ and 4 torsional modes (${}_2t_0$, ${}_3t_0$, ${}_4t_0$) for our set of 34 stellar models. The Newtonian first-order rotational corrections $C_{1,\text{New}}$ are equal to $1/[\ell(\ell+1)]$ for all stellar models according to Strohmayer’s formula (70). As expected, for each EoS, higher compactnesses M/R result in higher relativistic corrections rc_r . For the chosen set of stellar models, where $0.14 \lesssim M/R \lesssim 0.28$, the relativistic corrections rc_r vary from 10% to 30% approximately. According to equation (74) and for $\ell = 2$ ($C_{1,\text{New}} = 0.167$), an inertial observer measures relativistic corrections $rc_i = -rc_r/5$, that is from -2% to -6% approximately. These results are in good agreement with earlier investigations (Vavoulidis et al. 2007). In Table 2 we turn our attention to higher ($n = 1, 2, 3$) torsional overtones. Dipole ($\ell = 1$) modes are now allowed by the angular-momentum conservation law and we additionally know that these higher overtones are quite insensitive to the azimuthal harmonic index ℓ (Hansen & Cioffi 1980; McDermott, Van Horn & Hansen 1988). Table 2 refers to $\ell = 1$ but results for $\ell \gtrsim 2$ are very much alike. In Appendix B, we briefly show that the relativistic first-order rotational corrections of the eigenfrequencies of the torsional modes can be estimated also by the integral formula:

$$\sigma_1 = m\Omega C_{1,\text{rel}} = m \frac{\int_0^R \varpi [1/\Lambda + \varepsilon v_s^2 (1 - 2/\Lambda)] (T^0)^2 dr}{\int_0^R (T^0)^2 dr}. \quad (75)$$

The numerical evaluation of formula (75) and the numerical solution of the previously described eigenvalue problem yield essentially the same results for σ_1 .

Finally, in Figure 1 we show the relativistic corrections rc_r versus compactnesses M/R for our set of 34 stellar models, based on the results of Table 1. Clearly, models with the NV crustal EoS are shifted towards higher rc_r in respect with models with the DH crustal EoS; linear fits reveal that:

$$rc_r (\%) \simeq 108.9 (\pm 3.0) M/R - 3.5 (\pm 0.7), \quad \text{for the DH crustal EoS}, \quad (76)$$

$$rc_r (\%) \simeq 108.9 (\pm 4.9) M/R - 2.5 (\pm 1.1), \quad \text{for the NV crustal EoS}, \quad (77)$$

Table 1. Eigenfrequencies (σ_0), Newtonian first-order rotational corrections ($C_{1,\text{New}}$) and relativistic first-order rotational corrections ($C_{1,\text{rel}}$) of the $\ell = 2, 3$ and 4 , $n = 0$ **torsional modes** (${}_2t_0$, ${}_3t_0$, ${}_4t_0$) for various stellar models. As expected, for each EoS, higher compactnesses (M/R) result in higher relativistic corrections ($r_{\text{cr}} := 1 - C_{1,\text{rel}}/C_{1,\text{New}}$). The Newtonian first-order rotational corrections are equal to $1/[\ell(\ell+1)]$ for all stellar models.

| Model | M/R | σ_0 (Hz) | $\ell = 2$ | | σ_0 (Hz) | $\ell = 3$ | | σ_0 (Hz) | $\ell = 4$ | | r_{cr} (%) |
|-----------------------|-------|-----------------|--------------------|--------------------|-----------------|--------------------|--------------------|-----------------|--------------------|--------------------|---------------------|
| | | | $C_{1,\text{New}}$ | $C_{1,\text{rel}}$ | | $C_{1,\text{New}}$ | $C_{1,\text{rel}}$ | | $C_{1,\text{New}}$ | $C_{1,\text{rel}}$ | |
| A+DH ₁₄ | 0.218 | 28.4 | 0.167 | 0.134 | 44.9 | 0.083 | 0.067 | 60.3 | 0.05 | 0.040 | 19.55 |
| A+DH ₁₆ | 0.264 | 27.1 | 0.167 | 0.125 | 42.9 | 0.083 | 0.063 | 57.5 | 0.05 | 0.038 | 24.89 |
| WFF3+DH ₁₄ | 0.191 | 26.3 | 0.167 | 0.139 | 41.6 | 0.083 | 0.069 | 55.7 | 0.05 | 0.042 | 16.81 |
| WFF3+DH ₁₆ | 0.223 | 25.2 | 0.167 | 0.133 | 39.8 | 0.083 | 0.067 | 53.4 | 0.05 | 0.040 | 20.16 |
| WFF3+DH ₁₈ | 0.265 | 24.2 | 0.167 | 0.125 | 38.3 | 0.083 | 0.062 | 51.3 | 0.05 | 0.037 | 25.07 |
| APR+DH ₁₄ | 0.171 | 24.6 | 0.167 | 0.142 | 38.8 | 0.083 | 0.071 | 52.1 | 0.05 | 0.043 | 14.85 |
| APR+DH ₁₆ | 0.195 | 23.3 | 0.167 | 0.138 | 36.9 | 0.083 | 0.069 | 49.5 | 0.05 | 0.041 | 17.40 |
| APR+DH ₁₈ | 0.221 | 22.2 | 0.167 | 0.133 | 35.1 | 0.083 | 0.066 | 47.1 | 0.05 | 0.040 | 20.26 |
| APR+DH ₂₀ | 0.248 | 21.2 | 0.167 | 0.127 | 33.5 | 0.083 | 0.064 | 45.0 | 0.05 | 0.038 | 23.58 |
| APR+DH ₂₂ | 0.279 | 20.1 | 0.167 | 0.120 | 31.8 | 0.083 | 0.060 | 42.6 | 0.05 | 0.036 | 27.76 |
| L+DH ₁₄ | 0.141 | 21.5 | 0.167 | 0.146 | 34.0 | 0.083 | 0.073 | 45.6 | 0.05 | 0.044 | 12.63 |
| L+DH ₁₆ | 0.160 | 20.5 | 0.167 | 0.143 | 32.5 | 0.083 | 0.071 | 43.6 | 0.05 | 0.043 | 14.38 |
| L+DH ₁₈ | 0.179 | 19.6 | 0.167 | 0.140 | 31.0 | 0.083 | 0.070 | 41.6 | 0.05 | 0.042 | 16.24 |
| L+DH ₂₀ | 0.199 | 18.9 | 0.167 | 0.136 | 29.9 | 0.083 | 0.068 | 40.1 | 0.05 | 0.041 | 18.28 |
| L+DH ₂₂ | 0.221 | 18.1 | 0.167 | 0.132 | 28.7 | 0.083 | 0.066 | 38.5 | 0.05 | 0.040 | 20.56 |
| L+DH ₂₄ | 0.244 | 17.5 | 0.167 | 0.128 | 27.7 | 0.083 | 0.064 | 37.1 | 0.05 | 0.038 | 23.24 |
| L+DH ₂₆ | 0.272 | 16.9 | 0.167 | 0.122 | 26.7 | 0.083 | 0.061 | 35.8 | 0.05 | 0.037 | 26.73 |
| | | | | | | | | | | | |
| A+NV ₁₄ | 0.218 | 28.7 | 0.167 | 0.133 | 45.4 | 0.083 | 0.066 | 60.9 | 0.05 | 0.040 | 20.24 |
| A+NV ₁₆ | 0.264 | 27.4 | 0.167 | 0.124 | 43.3 | 0.083 | 0.062 | 58.1 | 0.05 | 0.037 | 25.52 |
| WFF3+NV ₁₄ | 0.191 | 26.7 | 0.167 | 0.138 | 42.2 | 0.083 | 0.069 | 56.6 | 0.05 | 0.041 | 17.48 |
| WFF3+NV ₁₆ | 0.223 | 25.4 | 0.167 | 0.132 | 40.2 | 0.083 | 0.066 | 53.9 | 0.05 | 0.040 | 20.78 |
| WFF3+NV ₁₈ | 0.265 | 24.4 | 0.167 | 0.124 | 38.6 | 0.083 | 0.062 | 51.7 | 0.05 | 0.037 | 25.64 |
| APR+NV ₁₄ | 0.173 | 25.2 | 0.167 | 0.140 | 39.8 | 0.083 | 0.070 | 53.4 | 0.05 | 0.042 | 16.04 |
| APR+NV ₁₆ | 0.198 | 23.8 | 0.167 | 0.136 | 37.6 | 0.083 | 0.068 | 50.5 | 0.05 | 0.041 | 18.56 |
| APR+NV ₁₈ | 0.223 | 22.6 | 0.167 | 0.131 | 35.7 | 0.083 | 0.066 | 47.9 | 0.05 | 0.039 | 21.37 |
| APR+NV ₂₀ | 0.250 | 21.4 | 0.167 | 0.126 | 33.9 | 0.083 | 0.063 | 45.5 | 0.05 | 0.038 | 24.64 |
| APR+NV ₂₂ | 0.280 | 20.3 | 0.167 | 0.119 | 32.1 | 0.083 | 0.059 | 43.1 | 0.05 | 0.036 | 28.75 |
| L+NV ₁₄ | 0.152 | 23.2 | 0.167 | 0.142 | 36.6 | 0.083 | 0.071 | 49.2 | 0.05 | 0.043 | 15.01 |
| L+NV ₁₆ | 0.171 | 21.8 | 0.167 | 0.139 | 34.5 | 0.083 | 0.069 | 46.3 | 0.05 | 0.042 | 16.86 |
| L+NV ₁₈ | 0.190 | 20.7 | 0.167 | 0.135 | 32.7 | 0.083 | 0.068 | 43.9 | 0.05 | 0.041 | 18.78 |
| L+NV ₂₀ | 0.210 | 19.7 | 0.167 | 0.132 | 31.1 | 0.083 | 0.066 | 41.8 | 0.05 | 0.040 | 20.85 |
| L+NV ₂₂ | 0.230 | 18.8 | 0.167 | 0.128 | 29.7 | 0.083 | 0.064 | 39.8 | 0.05 | 0.038 | 23.12 |
| L+NV ₂₄ | 0.253 | 18.0 | 0.167 | 0.124 | 28.4 | 0.083 | 0.062 | 38.1 | 0.05 | 0.037 | 25.77 |
| L+NV ₂₆ | 0.281 | 17.2 | 0.167 | 0.118 | 27.2 | 0.083 | 0.059 | 36.5 | 0.05 | 0.035 | 29.24 |

and these can be combined with the fitting formulas (68)-(69) of the previous Subsection.

3.3 Interfacial and shear modes

The Newtonian first-order rotational corrections of the eigenfrequencies of the interfacial and shear modes can not be given by an analytic formula like Strohmayer's formula (70). However, these corrections are given by the integral formula (Unno et al. 1989; Strohmayer 1991; Lee & Strohmayer 1996):

$$C_{1,\text{New}} = \frac{\int_0^R \left[2\bar{\xi}^r \bar{\xi}^h + (\bar{\xi}^h)^2 \right] \rho r^2 dr}{\int_0^R \left[(\bar{\xi}^r)^2 + \ell(\ell+1)(\bar{\xi}^h)^2 \right] \rho r^2 dr}, \quad (78)$$

where $\bar{\xi}^r = rS^0$, $\bar{\xi}^h = rH^0$ are the zeroth-order radial eigenfunctions of the displacement vector (compare with equation 9). We clearly see that the Newtonian first-order rotational corrections of interfacial and shear spheroidal modes do depend on the background stellar model whereas torsional toroidal modes were proved to be independent.

In Table 3 we list values of C_1 for the $\ell = 1, 2$ and 3 interfacial modes for our set of 34 stellar models. $C_{1,\text{New}}$ is calculated by evaluating the integral formula (78) while $C_{1,\text{rel}}$ is calculated by solving the eigenvalue problem (34)-(37), (54)-(55) along with the appropriate boundary and jump conditions. In both cases, we first need to solve the zeroth-order eigenvalue problem in order to determine the zeroth-order eigenfrequency and the zeroth-order eigenfunctions.

Table 2. Eigenfrequencies (σ_0), Newtonian first-order rotational corrections ($C_{1,\text{New}}$) and relativistic first-order rotational corrections ($C_{1,\text{rel}}$) of the $\ell = 1$, $n = 1, 2$ and 3 **torsional modes** (${}_1t_1$, ${}_1t_2$, ${}_1t_3$) for various stellar models. As expected, for each EoS, higher compactnesses (M/R) result in higher relativistic corrections ($r_{\text{cr}} := 1 - C_{1,\text{rel}}/C_{1,\text{New}}$). The Newtonian first-order rotational corrections are equal to $1/2$ for all stellar models as $\ell = 1$.

| Model | M/R | σ_0 (Hz) | $n = 1$ | | σ_0 (Hz) | $n = 2$ | | σ_0 (Hz) | $n = 3$ | | r_{cr} (%) |
|-----------------------|-------|-----------------|--------------------|--------------------|-----------------|--------------------|--------------------|-----------------|--------------------|--------------------|---------------------|
| | | | $C_{1,\text{New}}$ | $C_{1,\text{rel}}$ | | $C_{1,\text{New}}$ | $C_{1,\text{rel}}$ | | $C_{1,\text{New}}$ | $C_{1,\text{rel}}$ | |
| A+DH ₁₄ | 0.218 | 1203.9 | 0.5 | 0.406 | 2009.4 | 0.5 | 0.406 | 2775.7 | 0.5 | 0.406 | 18.89 |
| A+DH ₁₆ | 0.264 | 1527.6 | 0.5 | 0.379 | 2549.4 | 0.5 | 0.379 | 3523.9 | 0.5 | 0.379 | 24.30 |
| WFF3+DH ₁₄ | 0.191 | 940.7 | 0.5 | 0.419 | 1570.6 | 0.5 | 0.420 | 2165.4 | 0.5 | 0.420 | 16.11 |
| WFF3+DH ₁₆ | 0.223 | 1099.0 | 0.5 | 0.402 | 1834.0 | 0.5 | 0.403 | 2531.8 | 0.5 | 0.403 | 19.50 |
| WFF3+DH ₁₈ | 0.265 | 1363.8 | 0.5 | 0.378 | 2275.3 | 0.5 | 0.378 | 3144.0 | 0.5 | 0.378 | 24.48 |
| APR+DH ₁₄ | 0.171 | 760.0 | 0.5 | 0.429 | 1268.9 | 0.5 | 0.429 | 1748.2 | 0.5 | 0.430 | 14.12 |
| APR+DH ₁₆ | 0.195 | 858.7 | 0.5 | 0.417 | 1433.5 | 0.5 | 0.417 | 1976.6 | 0.5 | 0.417 | 16.69 |
| APR+DH ₁₈ | 0.221 | 963.7 | 0.5 | 0.402 | 1608.4 | 0.5 | 0.402 | 2219.7 | 0.5 | 0.403 | 19.59 |
| APR+DH ₂₀ | 0.248 | 1080.9 | 0.5 | 0.385 | 1803.4 | 0.5 | 0.385 | 2490.8 | 0.5 | 0.386 | 22.95 |
| APR+DH ₂₂ | 0.279 | 1234.7 | 0.5 | 0.364 | 2060.1 | 0.5 | 0.364 | 2846.4 | 0.5 | 0.365 | 27.17 |
| L+DH ₁₄ | 0.141 | 529.2 | 0.5 | 0.441 | 884.2 | 0.5 | 0.441 | 1217.4 | 0.5 | 0.442 | 11.83 |
| L+DH ₁₆ | 0.160 | 585.3 | 0.5 | 0.432 | 977.7 | 0.5 | 0.432 | 1347.9 | 0.5 | 0.433 | 13.61 |
| L+DH ₁₈ | 0.179 | 647.0 | 0.5 | 0.423 | 1080.6 | 0.5 | 0.423 | 1490.7 | 0.5 | 0.423 | 15.50 |
| L+DH ₂₀ | 0.199 | 711.2 | 0.5 | 0.412 | 1187.4 | 0.5 | 0.412 | 1639.6 | 0.5 | 0.413 | 17.57 |
| L+DH ₂₂ | 0.221 | 786.6 | 0.5 | 0.401 | 1313.2 | 0.5 | 0.401 | 1814.0 | 0.5 | 0.401 | 19.88 |
| L+DH ₂₄ | 0.244 | 872.3 | 0.5 | 0.387 | 1456.0 | 0.5 | 0.387 | 2012.5 | 0.5 | 0.388 | 22.59 |
| L+DH ₂₆ | 0.272 | 992.6 | 0.5 | 0.369 | 1656.3 | 0.5 | 0.369 | 2290.8 | 0.5 | 0.370 | 26.13 |
| | | | | | | | | | | | |
| A+NV ₁₄ | 0.218 | 950.5 | 0.5 | 0.402 | 1687.5 | 0.5 | 0.402 | 2390.0 | 0.5 | 0.403 | 19.68 |
| A+NV ₁₆ | 0.264 | 1190.4 | 0.5 | 0.375 | 2113.3 | 0.5 | 0.375 | 2997.5 | 0.5 | 0.376 | 25.03 |
| WFF3+NV ₁₄ | 0.191 | 740.2 | 0.5 | 0.415 | 1314.7 | 0.5 | 0.416 | 1860.4 | 0.5 | 0.417 | 16.90 |
| WFF3+NV ₁₆ | 0.223 | 865.4 | 0.5 | 0.399 | 1536.4 | 0.5 | 0.399 | 2176.3 | 0.5 | 0.400 | 20.24 |
| WFF3+NV ₁₈ | 0.265 | 1069.5 | 0.5 | 0.374 | 1898.0 | 0.5 | 0.374 | 2691.4 | 0.5 | 0.375 | 25.14 |
| APR+NV ₁₄ | 0.173 | 615.8 | 0.5 | 0.423 | 1094.2 | 0.5 | 0.423 | 1546.9 | 0.5 | 0.424 | 15.43 |
| APR+NV ₁₆ | 0.198 | 688.0 | 0.5 | 0.410 | 1222.1 | 0.5 | 0.410 | 1730.2 | 0.5 | 0.411 | 17.98 |
| APR+NV ₁₈ | 0.223 | 769.0 | 0.5 | 0.396 | 1365.3 | 0.5 | 0.396 | 1934.3 | 0.5 | 0.397 | 20.81 |
| APR+NV ₂₀ | 0.250 | 857.9 | 0.5 | 0.379 | 1523.0 | 0.5 | 0.380 | 2159.3 | 0.5 | 0.380 | 24.11 |
| APR+NV ₂₂ | 0.280 | 974.1 | 0.5 | 0.359 | 1728.6 | 0.5 | 0.359 | 2451.9 | 0.5 | 0.359 | 28.26 |
| L+NV ₁₄ | 0.152 | 483.0 | 0.5 | 0.428 | 858.5 | 0.5 | 0.429 | 1212.3 | 0.5 | 0.430 | 14.35 |
| L+NV ₁₆ | 0.171 | 524.0 | 0.5 | 0.419 | 931.1 | 0.5 | 0.419 | 1316.6 | 0.5 | 0.420 | 16.21 |
| L+NV ₁₈ | 0.190 | 567.2 | 0.5 | 0.409 | 1007.6 | 0.5 | 0.409 | 1426.1 | 0.5 | 0.410 | 18.16 |
| L+NV ₂₀ | 0.210 | 614.5 | 0.5 | 0.399 | 1091.3 | 0.5 | 0.399 | 1545.8 | 0.5 | 0.400 | 20.25 |
| L+NV ₂₂ | 0.230 | 666.7 | 0.5 | 0.387 | 1183.9 | 0.5 | 0.387 | 1678.1 | 0.5 | 0.388 | 22.55 |
| L+NV ₂₄ | 0.253 | 728.8 | 0.5 | 0.374 | 1294.0 | 0.5 | 0.374 | 1835.2 | 0.5 | 0.375 | 25.23 |
| L+NV ₂₆ | 0.281 | 823.8 | 0.5 | 0.356 | 1462.1 | 0.5 | 0.356 | 2074.1 | 0.5 | 0.357 | 28.74 |

In Table 4 we focus on the shear modes. These eigenmodes are quite insensitive to the value of ℓ , thus we fix $\ell = 1$ and list the Newtonian and the relativistic first-order rotational corrections of the first three members of this family of modes (${}_1s_1$, ${}_1s_2$, ${}_1s_3$). We note that the eigenfrequencies of the shear modes are almost equal to those of the higher overtones of the torsional modes. This is expected as s-modes and t-modes represent different polarizations of transverse elastic waves.

4 DISCUSSION

In this paper, we have calculated the first-order rotational corrections of the torsional, interfacial and shear modes of realistic neutron stars with crusts within the framework of General Relativity. Our results agree, qualitatively, with previous Newtonian results but differ, quantitatively, a few per cent. An interesting observation is that, although the low-frequency torsional modes could not explain all of the 18, 26 and 30 Hz frequencies observed in SGRs, there is a good chance that one or two of them could be explained by low-frequency interfacial modes as it can be seen from Table 3. This would be a novel scenario for gravitational-wave detection since it would imply the excitation of polar-type modes via their coupling with torsional oscillations.

Rotation could drive many oscillation modes unstable through the Chandrasekhar-Friedman-Schutz (CFS) mechanism. For an observer rotating with the star, the instability would set in when the eigenfrequency of a mode σ_r would be equal to $m\Omega$; at the same time, the eigenfrequency of the same mode for a distant inertial observer σ_i would be zero. For the torsional and the

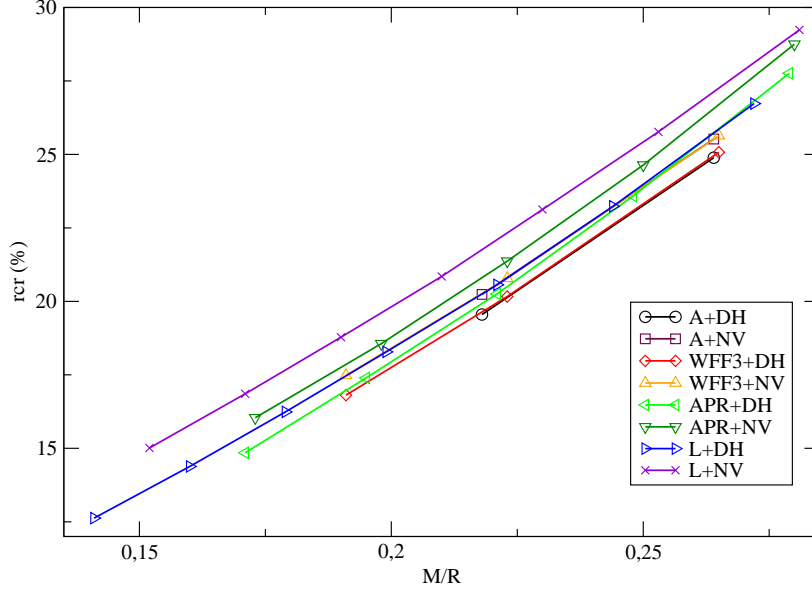


Figure 1. Relativistic correction as measured from a rotating observer (rcr) versus compactness (M/R) for the fundamental torsional modes (${}_2t_0$) for various EoS.

interfacial crustal modes this would happen at moderate stellar rotational frequencies (Yoshida & Lee 2001; Vavoulidis et al. 2007). For example, in Figure 2, which refers to the L+DH₁₄ EoS, we show that this would happen at $\Omega \simeq 12.5\text{Hz}$ for the torsional ${}_2t_0$ mode and at $\Omega \simeq 19\text{Hz}$ for the interfacial ${}_2i$ mode. We have adopted the slow-rotation approximation but this proves to be sufficient in most cases. For example, SGRs rotate with periods of $\sim 5\text{--}8$ secs (Woods & Thompson 2006) which are extremely high compared to their Kepler periods ($\gtrsim 1\text{ms}$). On the other hand, for such slow rotators, rotational effects are quite small; it would be a big surprise if they could be measured from current x-ray detectors. Furthermore, CFS instabilities of crustal modes could not be developed in those objects as they would require stellar rotational frequencies of several tenths to hundreds of Hz to set in. However, for newly-born neutron stars things may be quite different. Newly-born neutron stars are expected to rotate much faster, with frequencies from tenths to hundreds of Hz, and therefore they could become CFS unstable soon after they would form their crusts. In such a case, the slow-rotation approximation would be still valid while the effects of rotation in the spectrum would become pronounced.

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Table 3. Eigenfrequencies (σ_0), Newtonian first-order rotational corrections ($C_{1,\text{New}}$) and relativistic first-order rotational corrections ($C_{1,\text{rel}}$) of the $\ell = 1, 2$ and 3 **interfacial modes** (${}_1i$, ${}_2i$, ${}_3i$) for various stellar models.

| Model | σ_0 (Hz) | $\ell = 1$ | | σ_0 (Hz) | $\ell = 2$ | | σ_0 (Hz) | $\ell = 3$ | |
|-----------------------|-----------------|--------------------|--------------------|-----------------|--------------------|--------------------|-----------------|--------------------|--------------------|
| | | $C_{1,\text{New}}$ | $C_{1,\text{rel}}$ | | $C_{1,\text{New}}$ | $C_{1,\text{rel}}$ | | $C_{1,\text{New}}$ | $C_{1,\text{rel}}$ |
| A+DH ₁₄ | 22.5 | 0.485 | 0.393 | 48.0 | 0.152 | 0.125 | 70.5 | 0.069 | 0.059 |
| A+DH ₁₆ | 19.4 | 0.489 | 0.370 | 43.1 | 0.156 | 0.120 | 63.7 | 0.073 | 0.058 |
| WFF3+DH ₁₄ | 18.1 | 0.482 | 0.403 | 40.7 | 0.150 | 0.127 | 60.1 | 0.067 | 0.059 |
| WFF3+DH ₁₆ | 17.7 | 0.486 | 0.390 | 39.4 | 0.153 | 0.125 | 58.2 | 0.070 | 0.059 |
| WFF3+DH ₁₈ | 17.1 | 0.490 | 0.369 | 38.1 | 0.157 | 0.120 | 56.3 | 0.074 | 0.058 |
| APR+DH ₁₄ | 19.9 | 0.480 | 0.410 | 42.0 | 0.148 | 0.128 | 61.2 | 0.066 | 0.059 |
| APR+DH ₁₆ | 17.4 | 0.484 | 0.401 | 37.8 | 0.151 | 0.127 | 55.5 | 0.069 | 0.059 |
| APR+DH ₁₈ | 16.7 | 0.487 | 0.390 | 36.3 | 0.154 | 0.125 | 53.4 | 0.071 | 0.059 |
| APR+DH ₂₀ | 18.0 | 0.489 | 0.375 | 37.5 | 0.156 | 0.122 | 54.8 | 0.073 | 0.059 |
| APR+DH ₂₂ | 15.1 | 0.491 | 0.357 | 32.9 | 0.158 | 0.117 | 48.5 | 0.075 | 0.057 |
| L+DH ₁₄ | 15.1 | 0.479 | 0.419 | 33.3 | 0.147 | 0.130 | 48.7 | 0.065 | 0.059 |
| L+DH ₁₆ | 15.9 | 0.482 | 0.413 | 33.9 | 0.150 | 0.130 | 49.5 | 0.067 | 0.060 |
| L+DH ₁₈ | 13.3 | 0.485 | 0.406 | 29.9 | 0.152 | 0.129 | 44.1 | 0.069 | 0.060 |
| L+DH ₂₀ | 14.7 | 0.487 | 0.399 | 31.4 | 0.154 | 0.128 | 46.0 | 0.071 | 0.060 |
| L+DH ₂₂ | 12.5 | 0.488 | 0.389 | 28.1 | 0.156 | 0.126 | 41.4 | 0.073 | 0.060 |
| L+DH ₂₄ | 13.1 | 0.490 | 0.378 | 28.4 | 0.157 | 0.123 | 41.8 | 0.074 | 0.059 |
| L+DH ₂₆ | 13.4 | 0.492 | 0.362 | 28.5 | 0.159 | 0.119 | 41.9 | 0.076 | 0.058 |
| | | | | | | | | | |
| A+NV ₁₄ | 20.7 | 0.480 | 0.385 | 45.4 | 0.147 | 0.121 | 66.7 | 0.065 | 0.056 |
| A+NV ₁₆ | 19.7 | 0.485 | 0.365 | 43.4 | 0.152 | 0.118 | 63.9 | 0.070 | 0.056 |
| WFF3+NV ₁₄ | 19.2 | 0.477 | 0.396 | 42.0 | 0.146 | 0.123 | 61.4 | 0.064 | 0.056 |
| WFF3+NV ₁₆ | 18.3 | 0.482 | 0.384 | 40.1 | 0.150 | 0.122 | 58.9 | 0.067 | 0.057 |
| WFF3+NV ₁₈ | 17.6 | 0.486 | 0.365 | 38.7 | 0.154 | 0.118 | 56.9 | 0.071 | 0.057 |
| APR+NV ₁₄ | 18.1 | 0.477 | 0.400 | 39.3 | 0.146 | 0.125 | 57.2 | 0.064 | 0.057 |
| APR+NV ₁₆ | 17.1 | 0.480 | 0.392 | 37.3 | 0.149 | 0.124 | 54.5 | 0.067 | 0.058 |
| APR+NV ₁₈ | 16.2 | 0.483 | 0.381 | 35.5 | 0.151 | 0.122 | 52.0 | 0.069 | 0.058 |
| APR+NV ₂₀ | 15.4 | 0.486 | 0.368 | 33.8 | 0.154 | 0.119 | 49.7 | 0.071 | 0.057 |
| APR+NV ₂₂ | 14.6 | 0.489 | 0.350 | 32.1 | 0.156 | 0.114 | 47.2 | 0.073 | 0.056 |
| L+NV ₁₄ | 16.6 | 0.479 | 0.406 | 35.9 | 0.147 | 0.126 | 52.1 | 0.064 | 0.057 |
| L+NV ₁₆ | 15.7 | 0.482 | 0.400 | 33.9 | 0.149 | 0.126 | 49.4 | 0.067 | 0.058 |
| L+NV ₁₈ | 14.8 | 0.484 | 0.393 | 32.2 | 0.152 | 0.125 | 47.1 | 0.069 | 0.058 |
| L+NV ₂₀ | 14.1 | 0.486 | 0.385 | 30.7 | 0.153 | 0.123 | 45.0 | 0.071 | 0.059 |
| L+NV ₂₂ | 13.4 | 0.488 | 0.375 | 29.4 | 0.155 | 0.122 | 43.1 | 0.072 | 0.058 |
| L+NV ₂₄ | 12.8 | 0.490 | 0.364 | 28.1 | 0.157 | 0.119 | 41.3 | 0.074 | 0.058 |
| L+NV ₂₆ | 12.3 | 0.491 | 0.349 | 27.0 | 0.158 | 0.115 | 39.7 | 0.075 | 0.057 |

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Table 4. Eigenfrequencies (σ_0), Newtonian first-order rotational corrections ($C_{1,\text{New}}$) and relativistic first-order rotational corrections ($C_{1,\text{rel}}$) of the $\ell = 1$, $n = 1, 2$ and 3 **shear modes** (${}_1s_1$, ${}_1s_2$, ${}_1s_3$) for various stellar models.

| Model | σ_0 (Hz) | $n = 1$ | | σ_0 (Hz) | $n = 2$ | | σ_0 (Hz) | $n = 3$ | |
|-----------------------|-----------------|--------------------|--------------------|-----------------|--------------------|--------------------|-----------------|--------------------|--------------------|
| | | $C_{1,\text{New}}$ | $C_{1,\text{rel}}$ | | $C_{1,\text{New}}$ | $C_{1,\text{rel}}$ | | $C_{1,\text{New}}$ | $C_{1,\text{rel}}$ |
| A+DH ₁₄ | 1203.7 | 0.492 | 0.399 | 2009.4 | 0.495 | 0.402 | 2775.7 | 0.496 | 0.403 |
| A+DH ₁₆ | 1527.4 | 0.494 | 0.375 | 2549.4 | 0.497 | 0.376 | 3523.9 | 0.497 | 0.377 |
| WFF3+DH ₁₄ | 940.6 | 0.490 | 0.411 | 1570.6 | 0.494 | 0.415 | 2165.4 | 0.495 | 0.416 |
| WFF3+DH ₁₆ | 1098.8 | 0.492 | 0.396 | 1833.9 | 0.496 | 0.399 | 2531.8 | 0.496 | 0.400 |
| WFF3+DH ₁₈ | 1363.7 | 0.494 | 0.374 | 2275.3 | 0.497 | 0.375 | 3144.0 | 0.497 | 0.376 |
| APR+DH ₁₄ | 759.8 | 0.488 | 0.419 | 1268.9 | 0.493 | 0.424 | 1748.2 | 0.493 | 0.425 |
| APR+DH ₁₆ | 858.5 | 0.490 | 0.408 | 1433.4 | 0.494 | 0.412 | 1976.5 | 0.495 | 0.413 |
| APR+DH ₁₈ | 963.6 | 0.492 | 0.396 | 1608.4 | 0.495 | 0.399 | 2219.7 | 0.496 | 0.400 |
| APR+DH ₂₀ | 1080.8 | 0.493 | 0.380 | 1803.3 | 0.496 | 0.383 | 2490.8 | 0.497 | 0.383 |
| APR+DH ₂₂ | 1234.6 | 0.495 | 0.361 | 2060.0 | 0.497 | 0.362 | 2846.4 | 0.497 | 0.363 |
| L+DH ₁₄ | 529.0 | 0.484 | 0.427 | 884.2 | 0.491 | 0.433 | 1217.4 | 0.491 | 0.434 |
| L+DH ₁₆ | 585.2 | 0.486 | 0.420 | 977.7 | 0.492 | 0.426 | 1347.9 | 0.493 | 0.427 |
| L+DH ₁₈ | 646.9 | 0.488 | 0.413 | 1080.5 | 0.494 | 0.417 | 1490.7 | 0.494 | 0.418 |
| L+DH ₂₀ | 711.1 | 0.490 | 0.404 | 1187.3 | 0.495 | 0.408 | 1639.6 | 0.495 | 0.409 |
| L+DH ₂₂ | 786.5 | 0.492 | 0.394 | 1313.1 | 0.496 | 0.397 | 1814.0 | 0.496 | 0.398 |
| L+DH ₂₄ | 872.3 | 0.493 | 0.382 | 1455.9 | 0.496 | 0.384 | 2012.5 | 0.497 | 0.385 |
| L+DH ₂₆ | 992.5 | 0.495 | 0.366 | 1656.3 | 0.497 | 0.367 | 2290.8 | 0.497 | 0.368 |
| | | | | | | | | | |
| A+NV ₁₄ | 950.2 | 0.492 | 0.395 | 1687.4 | 0.496 | 0.399 | 2389.9 | 0.496 | 0.400 |
| A+NV ₁₆ | 1190.2 | 0.494 | 0.371 | 2113.2 | 0.497 | 0.373 | 2997.5 | 0.498 | 0.374 |
| WFF3+NV ₁₄ | 739.9 | 0.490 | 0.407 | 1314.6 | 0.495 | 0.412 | 1860.3 | 0.495 | 0.413 |
| WFF3+NV ₁₆ | 865.2 | 0.492 | 0.392 | 1536.3 | 0.496 | 0.396 | 2176.2 | 0.496 | 0.397 |
| WFF3+NV ₁₈ | 1069.3 | 0.494 | 0.370 | 1898.0 | 0.497 | 0.372 | 2691.3 | 0.498 | 0.373 |
| APR+NV ₁₄ | 615.5 | 0.488 | 0.413 | 1094.0 | 0.494 | 0.418 | 1546.8 | 0.494 | 0.419 |
| APR+NV ₁₆ | 687.7 | 0.490 | 0.402 | 1222.0 | 0.495 | 0.406 | 1730.1 | 0.496 | 0.408 |
| APR+NV ₁₈ | 768.8 | 0.492 | 0.390 | 1365.2 | 0.496 | 0.393 | 1934.2 | 0.496 | 0.394 |
| APR+NV ₂₀ | 857.7 | 0.493 | 0.375 | 1522.9 | 0.497 | 0.377 | 2159.2 | 0.497 | 0.378 |
| APR+NV ₂₂ | 973.9 | 0.495 | 0.355 | 1728.5 | 0.502 | 0.360 | 2451.8 | 0.498 | 0.358 |
| L+NV ₁₄ | 482.6 | 0.486 | 0.416 | 858.3 | 0.493 | 0.422 | 1212.1 | 0.493 | 0.424 |
| L+NV ₁₆ | 523.7 | 0.488 | 0.408 | 931.0 | 0.494 | 0.414 | 1316.5 | 0.494 | 0.415 |
| L+NV ₁₈ | 567.0 | 0.489 | 0.400 | 1007.5 | 0.495 | 0.405 | 1426.0 | 0.495 | 0.407 |
| L+NV ₂₀ | 614.3 | 0.491 | 0.391 | 1091.3 | 0.496 | 0.395 | 1545.7 | 0.496 | 0.397 |
| L+NV ₂₂ | 666.6 | 0.492 | 0.381 | 1183.8 | 0.496 | 0.385 | 1678.0 | 0.497 | 0.386 |
| L+NV ₂₄ | 728.7 | 0.494 | 0.369 | 1293.9 | 0.497 | 0.372 | 1835.1 | 0.497 | 0.373 |
| L+NV ₂₆ | 823.7 | 0.495 | 0.353 | 1462.0 | 0.497 | 0.355 | 2074.1 | 0.498 | 0.356 |

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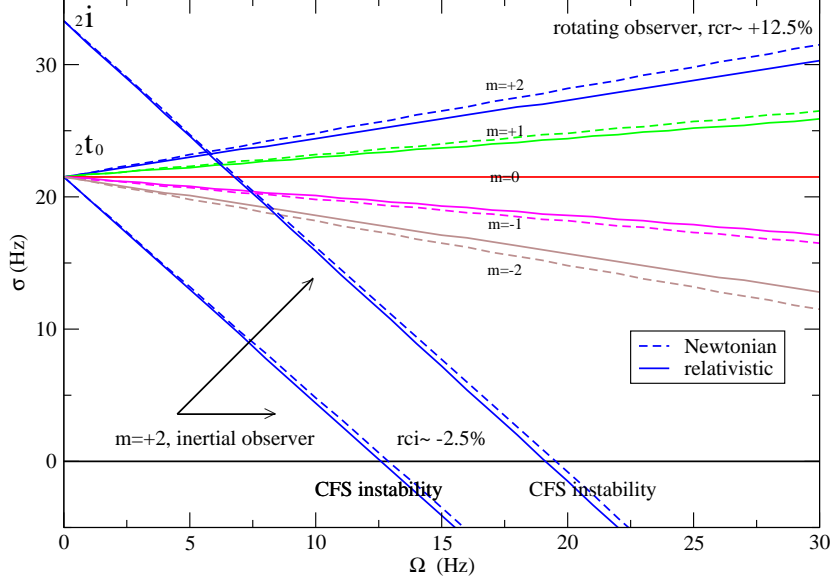


Figure 2. Eigenfrequency (σ) versus stellar rotational frequency (Ω) for the $\ell = 2$ interfacial ($2i$) and fundamental torsional ($2t_0$) modes for the stellar model L+DH14. Dashed lines correspond to Newtonian results while solid lines to relativistic ones. The relativistic corrections are generally small for an inertial observer, $rc_i \leq r_{cr}$. However, many low-frequency modes could be driven CFS unstable and General Relativity shifts the onset of the instability to set in at lower stellar rotational rates.

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APPENDIX A: RELATIVISTIC VERSION OF STROHMAYER'S (1991) EQUATIONS

Here we give the explicit form of the three second-order partial differential equations for the three components of the displacement vector $\xi^r = \xi^r(t, r, \theta, \phi)$, $\xi^\theta = \xi^\theta(t, r, \theta, \phi)$ and $\xi^\phi = \xi^\phi(t, r, \theta, \phi)$. In order to be able to compare easily with the Newtonian equations (35)-(39) of Strohmayer (1991), we define the displacement vector as $\xi^i = (\xi^r, \xi^\theta, \xi^\phi) := (\tilde{\xi}^r, \tilde{\xi}^\theta/r, \tilde{\xi}^\phi/r \sin \theta)$ and we get:

$$\begin{aligned}
 & - e^{2\varepsilon(\lambda-\nu)} \frac{\partial^2 \tilde{\xi}^r}{\partial t^2} + \varepsilon e^{-2\varepsilon\nu} c_s^2 r \varpi \frac{\partial^2 \tilde{\xi}^\phi}{\partial r \partial t} \sin \theta \\
 & + e^{-2\varepsilon\nu} \left((2 + \varepsilon c_s^2) \varpi + \varepsilon r \left(\frac{d\varpi}{dr} - \frac{d\nu}{dr} \varpi \right) (1 + \varepsilon c_s^2) - \varepsilon \Gamma r \frac{d\nu}{dr} \varpi \right) \frac{\partial \tilde{\xi}^\phi}{\partial t} \sin \theta \\
 & = \frac{\partial \chi}{\partial r} - \frac{A_r \Gamma p}{\rho + \varepsilon p} \alpha + \varepsilon c_s^2 \frac{d\nu}{dr} \frac{\partial \tilde{\xi}^r}{\partial r} + \varepsilon \left(c_s^2 \left(\frac{d^2 \nu}{dr^2} + \left(\frac{d\nu}{dr} \right)^2 \right) - (\Gamma - 1) \left(\frac{d\nu}{dr} \right)^2 \right) \tilde{\xi}^r \\
 & - \frac{\mu}{\rho + \varepsilon p} \left[\frac{1}{3} \frac{\partial \alpha}{\partial r} + 2\varepsilon \frac{d\lambda}{dr} \frac{\partial \tilde{\xi}^r}{\partial r} - \frac{2}{3} \left(\alpha - \varepsilon \left(\frac{d\nu}{dr} + 3 \frac{d\lambda}{dr} \right) \tilde{\xi}^r \right) \frac{1}{\mu} \frac{d\mu}{dr} + \frac{2}{\mu} \frac{d\mu}{dr} \frac{\partial \tilde{\xi}^r}{\partial r} + e^{2\varepsilon\lambda} \nabla^2 \tilde{\xi}^r \right. \\
 & - \left(\frac{2}{r^2} + \varepsilon \frac{1}{3} \frac{d^2 \nu}{dr^2} - \varepsilon \frac{d^2 \lambda}{dr^2} - \varepsilon \frac{4}{3} \frac{d\nu}{dr} \frac{d\lambda}{dr} + \varepsilon \frac{4}{3r} \frac{d\nu}{dr} - \varepsilon \frac{4}{r} \frac{d\lambda}{dr} \right) \tilde{\xi}^r - \left(\frac{2}{r^2} + \varepsilon \frac{2}{3r} \frac{d\nu}{dr} \right) \frac{\partial \tilde{\xi}^\theta}{\partial \theta} \\
 & \left. - \left(\frac{2 \cot \theta}{r^2} + \varepsilon \frac{2 \cot \theta}{3r} \frac{d\nu}{dr} \right) \tilde{\xi}^\theta - \left(\frac{2}{r^2 \sin \theta} + \varepsilon \frac{2}{3r \sin \theta} \frac{d\nu}{dr} \right) \frac{\partial \tilde{\xi}^\phi}{\partial \phi} \right]
 \end{aligned}$$

$$+ \varepsilon \frac{\mu}{\rho + \varepsilon p} e^{-2\varepsilon\nu} \left[\frac{2}{3} r \left(\frac{d\varpi}{dr} + \left(\frac{1}{\mu} \frac{d\mu}{dr} - \frac{d\nu}{dr} \right) \varpi \right) \frac{\partial \tilde{\xi}^\phi}{\partial t} \sin \theta + \frac{11}{3} \varpi \frac{\partial \tilde{\xi}^\phi}{\partial t} \sin \theta - \frac{1}{3} r \varpi \frac{\partial^2 \tilde{\xi}^\phi}{\partial r \partial t} \sin \theta - 2e^{2\varepsilon\lambda} \varpi \frac{\partial^2 \tilde{\xi}^r}{\partial \phi \partial t} \right], \quad (\text{A1})$$

$$\begin{aligned} & - e^{-2\varepsilon\nu} \frac{\partial^2 \tilde{\xi}^\theta}{\partial t^2} + \varepsilon e^{-2\varepsilon\nu} c_s^2 \varpi \frac{\partial^2 \tilde{\xi}^\phi}{\partial \theta \partial t} \sin \theta + e^{-2\varepsilon\nu} (2 + \varepsilon c_s^2) \varpi \frac{\partial \tilde{\xi}^\phi}{\partial t} \cos \theta = \frac{1}{r} \frac{\partial \chi}{\partial \theta} + \varepsilon c_s^2 \frac{d\nu}{dr} \frac{1}{r} \frac{\partial \tilde{\xi}^r}{\partial \theta} \\ & - \frac{\mu}{\rho + \varepsilon p} \left[\frac{1}{3r} \frac{\partial \alpha}{\partial \theta} + \frac{e^{-2\varepsilon\lambda}}{\mu} \frac{d\mu}{dr} \left(\frac{\partial \tilde{\xi}^\theta}{\partial r} + \frac{e^{2\varepsilon\lambda}}{r} \frac{\partial \tilde{\xi}^r}{\partial \theta} - \frac{\tilde{\xi}^\theta}{r} \right) + \nabla^2 \tilde{\xi}^\theta \right. \\ & + \left(\frac{2}{r^2} + \varepsilon \frac{2}{3r} \frac{d\nu}{dr} \right) \frac{\partial \tilde{\xi}^r}{\partial \theta} - \left(e^{-2\varepsilon\lambda} \left(\frac{2}{r^2} + \varepsilon \frac{1}{r} \frac{d\nu}{dr} - \varepsilon \frac{1}{r} \frac{d\lambda}{dr} \right) - \frac{1}{r^2} + \frac{\cos^2 \theta}{r^2 \sin^2 \theta} \right) \tilde{\xi}^\theta - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial \tilde{\xi}^\phi}{\partial \phi} \Big] \\ & + \varepsilon \frac{\mu}{\rho + \varepsilon p} e^{-2\varepsilon\nu} \left[\frac{11}{3} \varpi \frac{\partial \tilde{\xi}^\phi}{\partial t} \cos \theta - \frac{1}{3} \varpi \frac{\partial^2 \tilde{\xi}^\phi}{\partial \theta \partial t} \sin \theta - 2\varpi \frac{\partial^2 \tilde{\xi}^\theta}{\partial \phi \partial t} \right], \quad (\text{A2}) \end{aligned}$$

$$\begin{aligned} & - e^{-2\varepsilon\nu} \frac{\partial^2 \tilde{\xi}^\phi}{\partial t^2} - \varepsilon e^{-2\varepsilon\nu} r \varpi \frac{\partial \chi}{\partial t} \sin \theta + \varepsilon e^{-2\varepsilon\nu} c_s^2 \varpi \frac{\partial^2 \tilde{\xi}^\phi}{\partial \phi \partial t} - e^{-2\varepsilon\nu} 2\varpi \frac{\partial \tilde{\xi}^\theta}{\partial t} \cos \theta \\ & - e^{-2\varepsilon\nu} \left(2\varpi + \varepsilon r \left(\frac{d\varpi}{dr} - 2 \frac{d\nu}{dr} \varpi \right) + \varepsilon c_s^2 r \frac{d\nu}{dr} \varpi \right) \frac{\partial \tilde{\xi}^r}{\partial t} \sin \theta = \frac{1}{r \sin \theta} \frac{\partial \chi}{\partial \phi} + \varepsilon c_s^2 \frac{d\nu}{dr} \frac{1}{r \sin \theta} \frac{\partial \tilde{\xi}^r}{\partial \phi} \\ & - \frac{\mu}{\rho + \varepsilon p} \left[\frac{1}{3r \sin \theta} \frac{\partial \alpha}{\partial \phi} + \frac{e^{-2\varepsilon\lambda}}{\mu} \frac{d\mu}{dr} \left(\frac{\partial \tilde{\xi}^\phi}{\partial r} + \frac{e^{2\varepsilon\lambda}}{r \sin \theta} \frac{\partial \tilde{\xi}^r}{\partial \phi} - \frac{\tilde{\xi}^\phi}{r} \right) + \nabla^2 \tilde{\xi}^\phi \right. \\ & + \left(\frac{2}{r^2 \sin \theta} + \varepsilon \frac{2}{3r \sin \theta} \frac{d\nu}{dr} \right) \frac{\partial \tilde{\xi}^r}{\partial \phi} - \left(e^{-2\varepsilon\lambda} \left(\frac{2}{r^2} + \varepsilon \frac{1}{r} \frac{d\nu}{dr} - \varepsilon \frac{1}{r} \frac{d\lambda}{dr} \right) - \frac{1}{r^2} + \frac{\cos^2 \theta}{r^2 \sin^2 \theta} \right) \tilde{\xi}^\phi + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial \tilde{\xi}^\theta}{\partial \phi} \Big] \\ & + \varepsilon \frac{\mu}{\rho + \varepsilon p} e^{-2\varepsilon\nu} \left[-r \left(\frac{d\varpi}{dr} + \left(\frac{1}{\mu} \frac{d\mu}{dr} - \frac{d\nu}{dr} + \frac{1}{3} \frac{d\lambda}{dr} \right) \varpi \right) \frac{\partial \tilde{\xi}^r}{\partial t} \sin \theta - \frac{14}{3} \varpi \frac{\partial \tilde{\xi}^r}{\partial t} \sin \theta - \frac{13}{3} \varpi \frac{\partial \tilde{\xi}^\theta}{\partial t} \cos \theta \right. \\ & \left. - \frac{1}{3} r \varpi \frac{\partial^2 \tilde{\xi}^r}{\partial r \partial t} \sin \theta - \frac{1}{3} \varpi \frac{\partial^2 \tilde{\xi}^\theta}{\partial \theta \partial t} \sin \theta - \frac{8}{3} \varpi \frac{\partial^2 \tilde{\xi}^\phi}{\partial \phi \partial t} \right], \quad (\text{A3}) \end{aligned}$$

where:

$$\chi := -\frac{\Gamma p}{\rho + \varepsilon p} \alpha - \frac{\tilde{\xi}^r}{\rho + \varepsilon p} \frac{dp}{dr} = -c_s^2 \alpha + \frac{d\nu}{dr} \tilde{\xi}^r, \quad (\text{A4})$$

$$\alpha := \nabla_i \xi^i = \frac{\partial \tilde{\xi}^r}{\partial r} + \left(\frac{2}{r} + \varepsilon \frac{d\nu}{dr} + \varepsilon \frac{d\lambda}{dr} \right) \tilde{\xi}^r + \frac{1}{r} \frac{\partial \tilde{\xi}^\theta}{\partial \theta} + \frac{\cot \theta}{r} \tilde{\xi}^\theta + \frac{1}{r \sin \theta} \frac{\partial \tilde{\xi}^\phi}{\partial \phi}, \quad (\text{A5})$$

$$\nabla^2 := \nabla_i \nabla^i = e^{-2\varepsilon\lambda} \left[\frac{\partial^2}{\partial r^2} + \left(\frac{2}{r} + \varepsilon \frac{d\nu}{dr} - \varepsilon \frac{d\lambda}{dr} \right) \frac{\partial}{\partial r} \right] + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}, \quad (\text{A6})$$

and where:

$$c_s^2 := \frac{\Gamma p}{\rho + \varepsilon p}, \quad (\text{A7})$$

is the speed of sound waves.

APPENDIX B: PROOF OF FORMULA (75)

Here we apply a mathematical technique that allows us to estimate the relativistic first-order rotational corrections of the eigenfrequencies of the torsional modes through a simple integral formula. The same technique has been applied in fluid slowly rotating relativistic stars by Yoshida & Kojima (1997).

Multiplying equation (14) with the conjugate function T^{0*} and integrating over the whole star we get:

$$\begin{aligned} & \int_0^R \left\{ v_s^2 e^{2\varepsilon(\nu-\lambda)} \left[\frac{d^2 T^1}{dr^2} + \left(\frac{4}{r} + \varepsilon \frac{d\nu}{dr} - \varepsilon \frac{d\lambda}{dr} + \frac{1}{\mu} \frac{d\mu}{dr} \right) \frac{dT^1}{dr} - e^{2\varepsilon\lambda} \frac{\Lambda - 2}{r^2} T^1 \right] + \sigma_0^2 T^1 \right\} T^{0*} dr \\ & + 2\sigma_0 \sigma_1 \int_0^R T^0 T^{0*} dr = 2m\sigma_0 \int_0^R \varpi \left[\frac{1}{\Lambda} + \varepsilon v_s^2 \left(1 - \frac{2}{\Lambda} \right) \right] T^0 T^{0*} dr. \quad (\text{B1}) \end{aligned}$$

But to zeroth order in Ω :

$$\int_0^R \left\{ v_s^2 e^{2\varepsilon(\nu-\lambda)} \left[\frac{d^2 T^0}{dr^2} + \left(\frac{4}{r} + \varepsilon \frac{d\nu}{dr} - \varepsilon \frac{d\lambda}{dr} + \frac{1}{\mu} \frac{d\mu}{dr} \right) \frac{dT^0}{dr} - e^{2\varepsilon\lambda} \frac{\Lambda - 2}{r^2} T^0 \right] + \sigma_0^2 T^0 \right\} T^{0*} dr = 0, \quad (\text{B2})$$

which implies that the first term in equation (B1) annuls, leaving the following integral formula for the corrections σ_1 :

$$\sigma_1 = m \frac{\int_0^R \varpi [1/\Lambda + \varepsilon v_s^2 (1 - 2/\Lambda)] (T^0)^2 dr}{\int_0^R (T^0)^2 dr}. \quad (\text{B3})$$

In the Newtonian limit ($\varepsilon \rightarrow 0, \varpi \rightarrow \Omega$), this formula becomes:

$$\sigma_1 = m\Omega (1/\Lambda) = \frac{m\Omega}{\ell(\ell+1)}, \quad (\text{B4})$$

as expected.